Quantum Stabilizers

CPSC 519 – Quantum Computation

Manjot Bal

UNIVERSITY OF CALGARY

Shor's 9-Qubit Code

9-Qubit Code

$$|0\rangle \rightarrow |\bar{0}\rangle = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$

$$|1\rangle \rightarrow |\bar{1}\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

Errors

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Stabilizers

$K|\psi\rangle = |\psi\rangle$

Let $\mathcal{E} = \{E_1 \otimes \ldots \otimes E_n | E_i \in \{X, Y, Z, I\}\}$
Let the quantum code Q be a subspace of $\mathbb{C}^{2^n}$.
Then, the stabilizer of Q is $S = \{M \in \mathcal{E} | Mv = v, \forall v \in Q\}$.

$$|0\rangle \rightarrow |\bar{0}\rangle = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$
$$|1\rangle \rightarrow |\bar{1}\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

M_1	Z	Z	I	I	I	I	I	I	I
M_2	Z	I	Z	I	I	I	I	I	I
M_3	I	I	I	Z	Z	I	I	I	I
M_4	I	I	I	Z	I	Z	I	I	I
M_5	I	I	I	I	I	I	Z	Z	I
M_6	I	I	I	I	I	I	Z	I	Z
M_7	X	X	X	X	X	X	I	I	I
M_8	X	X	X	I	I	I	X	X	X

UNIVERSITY OF CALGARY

Stabilizers

What about $M^* = I \otimes Z \otimes Z \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I$?

$$|0\rangle \rightarrow |\bar{0}\rangle = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$
$$|1\rangle \rightarrow |\bar{1}\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

M_1	Z	Z	I	I	I	I	I	I	I
M_2	Z	I	Z	I	I	I	I	I	I
M_3	I	I	I	Z	Z	I	I	I	I
M_4	I	I	I	Z	I	Z	I	I	I
M_5	I	I	I	I	I	I	Z	Z	I
M_6	I	I	I	I	I	I	Z	I	Z
M_7	X	X	X	X	X	X	I	I	I
M_8	X	X	X	I	I	I	X	X	X

UNIVERSITY OF CALGARY

Stabilizers

Let $\mathcal{E} = \{E_1 \otimes \ldots \otimes E_n | E_i \in \{X, Y, Z, I\}\}$
Let the quantum code Q be a subspace of $\mathbb{C}^{2^n}$.
Then, the stabilizer of Q is $S = \{M \in \mathcal{E} | Mv = v, \forall v \in Q\}$.

$M_i, M_j \in S$

$M_i M_j v = M_i v = v = M_j v = M_j M_i v$

M_1	Z	Z	I	I	I	I	I	I	I
M_2	Z	I	Z	I	I	I	I	I	I
M_3	I	I	I	Z	Z	I	I	I	I
M_4	I	I	I	Z	I	Z	I	I	I
M_5	I	I	I	I	I	I	Z	Z	I
M_6	I	I	I	I	I	I	Z	I	Z
M_7	X	X	X	X	X	X	I	I	I
M_8	X	X	X	I	I	I	X	X	X

UNIVERSITY OF CALGARY

Stabilizers

Let $\mathcal{E} = \{E_1 \otimes \ldots \otimes E_n | E_i \in \{X, Y, Z, I\}\}$
Let the quantum code Q be a subspace of $\mathbb{C}^{2^n}$.
Then, the stabilizer of Q is $S = \{M \in \mathcal{E} | Mv = v, \forall v \in Q\}$.

S contains 2^{n-k} elements assuming the minimal representation has $n-k$ generators

M_1	Z	Z	I	I	I	I	I	I	I
M_2	Z	I	Z	I	I	I	I	I	I
M_3	I	I	I	Z	Z	I	I	I	I
M_4	I	I	I	Z	I	Z	I	I	I
M_5	I	I	I	I	I	I	Z	Z	I
M_6	I	I	I	I	I	I	Z	I	Z
M_7	X	X	X	X	X	X	I	I	I
M_8	X	X	X	I	I	I	X	X	X

UNIVERSITY OF CALGARY

Stabilizers

Let $\mathcal{E} = \{E_1 \otimes \ldots \otimes E_n | E_i \in \{X, Y, Z, I\}\}$
Let the quantum code Q be a subspace of $\mathbb{C}^{2^n}$.
Then, the stabilizer of Q is $S = \{M \in \mathcal{E} | Mv = v, \forall v \in Q\}$.

If an error E anticommutes with some M in S then E is detectable.

e.g.

$E = X \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I \otimes I$

$E = I \otimes I \otimes I \otimes Z \otimes I \otimes I \otimes I \otimes I \otimes I$

$$|0\rangle \to |\bar{0}\rangle = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$
$$|1\rangle \to |\bar{1}\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

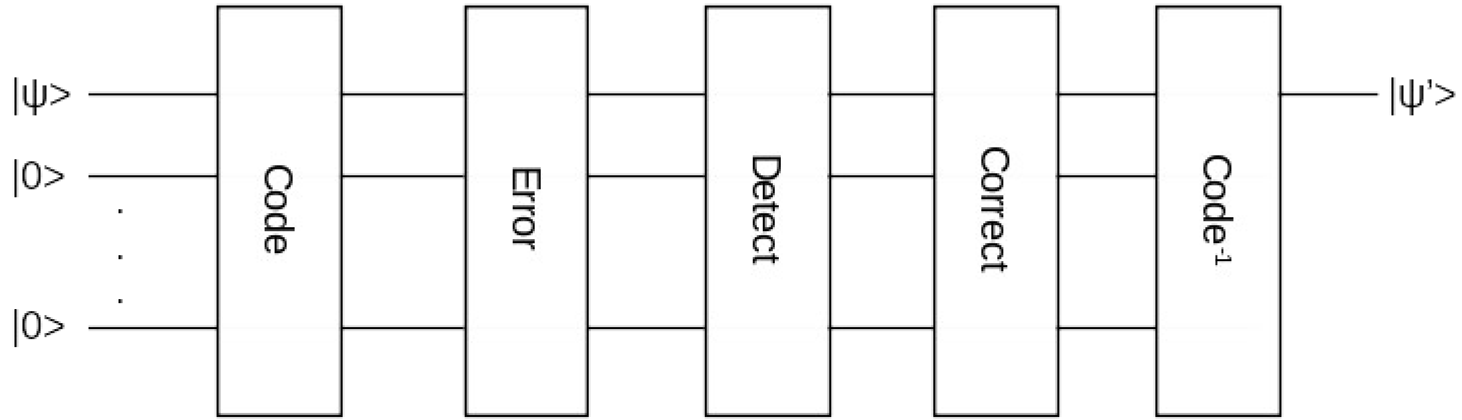
M_1	Z	Z	I	I	I	I	I	I	I
M_2	Z	I	Z	I	I	I	I	I	I
M_3	I	I	I	Z	Z	I	I	I	I
M_4	I	I	I	Z	I	Z	I	I	I
M_5	I	I	I	I	I	I	Z	Z	I
M_6	I	I	I	I	I	I	Z	I	Z
M_7	X	X	X	X	X	X	I	I	I
M_8	X	X	X	I	I	I	X	X	X

UNIVERSITY OF CALGARY

Stabilizers

- Simpler representation of a quantum code.
- Simpler to find the errors the code can detect and correct.
- We can build a circuit from the stabilizer.
- Reduces a multi-qubit state down to a single qubit state (assuming we are encoding one qubit).

Circuit



UNIVERSITY OF CALGARY

5-Qubit Code

- Shortest code to correct a single error.
- Non-CSS code.

M_1	X	Z	Z	X	I
M_2	I	X	Z	Z	X
M_3	X	I	X	Z	Z
M_4	Z	X	I	X	Z

$$
\begin{aligned}
|\bar{0}\rangle &= \sum_{M\in S} |00000\rangle \\
&= |00000\rangle + M_1|00000\rangle + M_2|00000\rangle + M_3|00000\rangle + M_4|00000\rangle \\
&+ M_1M_2|00000\rangle + M_1M_3|00000\rangle + M_1M_4|00000\rangle \\
&+ M_2M_3|00000\rangle + M_2M_4|00000\rangle + M_3M_4|00000\rangle \\
&+ M_1M_2M_3|00000\rangle + M_1M_2M_4|00000\rangle + M_1M_3M_4|00000\rangle + M_2M_3M_4|00000\rangle \\
&+ M_1M_2M_3M_4|00000\rangle
\end{aligned}
$$

$$|\bar{1}\rangle = X^{\otimes 5}|\bar{0}\rangle$$

UNIVERSITY OF CALGARY